THE EMPIRICAL CONTENT OF MODELS WITH SOCIAL INTERACTIONS

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Abstract. Empirical models with *social interactions* or *peer effects* allow the outcome of an individual to depend on the outcomes, choices, treatments, and/or characteristics of the other individuals in the group. We document the subtle relationship between the data and the objects of interest in models with interactions in small groups, and show that some econometric assumptions, that are direct extensions from models of individualistic treatment response, implicitly entail strong behavioral assumptions. We point out two such econometric assumptions, *EITR*, or *empirical individual treatment response,* and *EGTR*, or *empirical group treatment response*. In some cases EITR and/or EGTR are inconsistent with a class of plausible economic models for the interaction under consideration; in other cases these econometric assumptions imply significant assumptions on behavior that are not necessarily implied by economic theory. We illustrate this using relevant examples of interaction in immunization and disease, and in educational achievement. We conclude that it is important for applications in this class of models with small group interactions to recognize the restrictions some assumptions impose on behavior.

Keywords: small group social interaction, peer effects, treatment response model, social networks, best response function, equilibrium, linear-in-means model

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1. Introduction

Models of social interaction have seen considerable recent interest in both the theoretical and empirical literatures. The common theme to empirical models in this literature is to allow for a "interaction effect," or "peer effect" by which the outcome of any given individual is related to the outcomes, choices, treatments, and/or characteristics of the other individuals in its group.

It is typically assumed that each individual has a response function $\upsilon_{ig}(\cdot)$ that maps the choices, treatments and/or characteristics of individual i and the outcomes, choices, treatments, and/or characteristics of other individuals in its group to the outcome of individual i, in group g. For example, in the case of public health, $\upsilon_{ig}(\cdot)$ might give the health outcome of individual i as a function not only of whether individual i is vaccinated but also on whether the people in close contact to individual i are vaccinated, and perhaps other exogenous factors.[1] Alternatively, in the case of educational achievement, $\upsilon_{ig}(\cdot)$ might give the test score of student i as a function of the demographic characteristics of the students, some measure of teacher or school quality, and the test scores of the peers of student i.[2] More generally, this response function allows statements like: an individual whose peers have some characteristic is "caused" to also have that characteristic, rather than simply that there is some correlation in the data relating these characteristics.

The key methodological departure of this literature from the typical literature in econometrics is the possibility that the outcomes or choices of individual i are related to the outcomes or choices of the other individuals, and vice versa. This introduces a complicated simultaneity that does not exist in standard econometric models of individual choice and enlarges the scope of models analyzed to include richer models of economic behavior. This simultaneity also violates some of the main assumptions of the counterfactual treatment literature in statistics and biostatistics, especially SUTVA, or stable unit treatment value assumption, whereby, response of individual i depends on treatment given to that individual only. In the economic theory literature, on the other hand, simultaneity and interaction have been a key feature in most models with multiple decision makers.

[1] This corresponds to our notion of individual i's part of the group response, a notion that is defined below, because it expresses outcome only as a function of the exogenous variables.

[2] This corresponds to our notion of individual i's best response function, also a notion that is defined below, because it expresses outcome as a function of the outcomes of others, and the exogenous variables.

Models in the empirical literature with social interactions are typically estimated with data on many small groups of peers and this *small group* interaction is the exact setting we are interested in. Within each group the data consist of the outcome and treatment of each individual and perhaps other data like the demographic characteristics of the individuals. Among other assumptions, estimation of these models relies on two assumptions that we call *empirical individual treatment response* and *empirical group treatment response*. These assumptions are often times implicitly made and concern the relationship between the observed data and the underlying object of interest. The purpose of this paper is to point out that, in many settings of interest, these assumptions rule out interesting and relevant classes of economic models for the social interaction under consideration.

This paper is related to a large literature in empirical microeconomics that express the outcome of an individual as a function of the outcomes, choices, treatments, and/or characteristics of other individuals or peers[3]. These papers are typically agnostic about the kind of economic model under consideration. The motivation of early papers in this literature was to capture a sort of "spatial effect." See for example Cliff and Ord (1975), Case (1991), and Lee (2004). Manski (1993) studied a variant of the "linear-in-means" model in which the outcome of individual i depends on the population average outcome (and/or possibly other characteristics) of individual i's reference group, in a *"large group"* setting where it is assumed that the outcome of individuak i in the data, depends on the population conditional average of that outcome. The results in that paper essentially cast doubt on the identifiability of such a model. Subsequently, many papers revisited versions of this model and provided more positive results regarding its identification. Those variants write the model in terms of an individual's outcome depending on outcomes, choices, treatments, and/or characteristics of the finitely many members of that individual's reference group, or peers, which can be something like friends in a social network for example. See for example Sacerdote (2001), Krauth (2006), Graham (2008), Bramoulle, Djebbari, and Fortin (2009), and Lee, Liu, and Lin (2010) to name just a few. Similar to Manski (1993), Brock and Durlauf (2001) examine a binary response model with social interaction in large groups. There also, the outcome of an individual depends (nonlinearly) on the expectation of the outcome of others in that individual's large reference group. This is akin a game with incomplete information in large groups, whereas the behavior of

[3]A simple search for articles in the social sciences since year 2000 revealed 443 articles with "peer effects" in their titles, and more than 2000 articles with the words "social interaction" in their title.

an individual is directly linked to the whole group behavior and this group is large. This is in contrast to models of small group interaction that we consider here. We view the literature on inference in models with large group interaction as complementary to the kinds of models we study. For an in depth look at this literature of social interaction within large groups, see Blume, Brock, Durlauf, and Ioannides (2010). See also the recent book by Jackson (2010) that examines theoretical and empirical models of social and economic networks.

Section 2 discusses and formally states the two assumptions we study, and shows why these assumptions can be strong in models with social interaction. Section 3 discusses an example involving immunization and disease. Section 4 discusses an example involving educational achievement, and relates that to the linear-in-means model. Finally, the conclusion summarizes our results. Throughout the rest of the paper, and unless otherwise stated, by models with social interaction we mean models with social interactions in small groups.

2. Data and models with interaction

One possible object of interest in settings with social interaction is the best response function that determines the response of individual i in group g to the outcomes of the other individuals and the treatments in group g, denoted $\upsilon_{ig}(\mathbf{y}_{-ig}, \mathbf{d}_g)$. In the educational attainment example the best response function allows us to consider the effect on student i of some policy intervention that exogenously determines the educational attainment of its peers. Of course, it may not be plausible for a real world social planner to directly manipulate some individuals, but this counterfactual can still be useful to understand the social interaction effects that are present and is one way to justify claims like that peers "cause each other" to have certain outcomes, as in claims like that "a student who has high achieving peers has increased educational achievement itself." In other settings it is plausible that a social planner could directly manipulate some individuals, for example by preventing some actions for some individuals as in drug and alcohol policy, or by a mandatory immunization policy. More generally, the best response function can be useful to get a sense of the social interaction mechanism when considering a policy intervention that directly affects the outcomes of individuals privately, but because of the social interaction process may indirectly affect the outcomes of everyone in the group. The best response function can reveal the "partial" effect of the intervention that is due to the social interaction. Moreover, even if the effect of interest is not the social interaction effect directly, if a model does not allow the possibility of a social interaction effect, and such an effect is actually present, the model is mis-specified and so potentially the estimates of the effect of other treatments on outcomes are incorrect.

The assumption of *empirical individual treatment response* (EITR) is the assumption that the *observed data* $\mathbf{y}$ and $\mathbf{d}$ satisfy the equation $y_{ig} = \upsilon_{ig}(\mathbf{y}_{-ig}, \mathbf{d}_g)$. This assumption relates the economic model and associated objects of interest (i.e., the response functions $\upsilon_{ig}(\cdot)$) to the data generating process.[4] If there were no social interaction, then this assumption is standard and un-controversial: the response function at d for individual i is observed if i is treated with d. In contrast, the assumption of EITR in models with social interaction entails implicit behavioral assumptions that are not necessarily implied by economic theory, and in some cases are inconsistent with a class of economic models for the social interaction under consideration.

Another possible object of interest is the function (or correspondence) relating the vector of outcomes of all individuals (or, by implication, a subset of individuals) in the group to the exogenous treatments that can be plausibly manipulated by a social planner. (The outcomes $\mathbf{y}$ are determined endogenously by the model of social interaction, whereas the treatments $\mathbf{d}$ are exogenous to the model.) We call this the *group response correspondence* $\upsilon_g(\cdot)$, for group g. This is similar to an individualistic response function, but with vector outcomes and treatments. But unlike an individualistic response function, the social interaction presumably places some sort of additional structure on this object. It is key that we call this a correspondence because generically we should not assume that the social interaction process has a unique outcome. In contrast to this individual response function, the group response function $\upsilon_g(\cdot)$ does not reveal anything about the mechanism of the social interaction, but can be the object of interest when the question is the relationship between the exogenous covariates and the outcomes.

The assumption of *empirical group treatment response* (EGTR) is the assumption that $\mathbf{y}_g = \upsilon_g(\mathbf{d}_g)$ in the data, or equivalently that the mapping $\upsilon_g(\cdot)$ is a function. As with EITR, in models with social interaction this assumption involves implicit assumptions on behavior that are not necessarily consistent with, or implied by, economic theory. Instead, it must be assumed in general only that $\mathbf{y}_g \in \upsilon_g(\mathbf{d}_g)$, because

[4] In the individualistic treatment response literature this assumption basically translates to the assumption that the econometrician observes the outcome at a given treatment when that treatment is realized.

social interactions generically exhibit multiple potential outcomes, because of multiple equilibria and equilibria in mixed strategies with deliberate randomization.

This paper focuses on the case where the objects of interest are functionals of $\upsilon_{ig}(\cdot)$ and/or $\upsilon_g(\cdot)$; if, on the other hand, the objects of interest are summary statistics that simply characterize the data, not necessarily related to these response functions, our results do not apply. This paper focuses on the pair of assumptions we call empirical individual treatment response and empirical group treatment response. We formally define these as follows.

Definition 2.1. *Consider a model with best response functions $\upsilon_{ig}(\mathbf{y}_{-ig}, \mathbf{d}_g)$. The model is said to satisfy empirical individual treatment response, or EITR, if the data satisfy $y_{ig} = \upsilon_{ig}(\mathbf{y}_{-ig}, \mathbf{d}_g)$ for every individual i.*

Definition 2.2. *Consider a model with a group response function $\upsilon_g(\mathbf{d}_g)$. The model is said to satisfy empirical group treatment response, or EGTR, if the data satisfies $\mathbf{y}_g = \upsilon_g(\mathbf{d}_g)$ for every group g.*

In general, EITR and/or EGTR can be too strong in models with social interaction for at least four reasons.

First, EITR implicitly entails the assumption that the data consist of realizations of Nash equilibrium play, because it implies that the observed outcome of each individual i is the best response to the observed outcome of individuals $-i$. This is exactly the definition of Nash equilibrium play. It is well known that Nash equilibrium play often is not a reasonable characterization of actual behavior. Many alternatives to Nash equilibrium play are entertained both in the theoretical game theory literature and also the experimental literature. Among many other examples this includes rationalizability (i.e., Bernheim (1984) and Pearce (1984)), quantal response (i.e., McKelvey and Palfrey (1995)), and models of "bounded reasoning" (i.e., Camerer, Ho, and Chong (2004) and Costa-Gomes and Crawford (2006)).

Second, EITR implicitly entails the assumption of *pure strategy* Nash equilibrium play. This is for the same reason: if mixed strategies are used then it is possible even in Nash equilibrium play for the condition that $y_{ig} = \upsilon_{ig}(\mathbf{y}_{-ig}, \mathbf{d}_g)$ to be false, since with mixed strategies it is possible that the realizations of the mixed strategies do not (considered as pure strategies) comprise mutual best responses. In many games it is not reasonable to rule out mixed strategies, because many games do not have a Nash equilibrium in pure strategies, and in others it may be "reasonable" that when there are multiple equilibria the mixed strategy equilibrium is selected. For example, in some symmetric games the pure strategy Nash equilibria are asymmetric, and thus perhaps unreasonable[5]. Similarly, EGTR entails the assumption of pure strategies, although not necessarily in Nash equilibrium. This is because if there were mixed strategies with deliberate randomization, then there can be multiple potential outcomes even for a fixed vector of treatments $\mathbf{d}$.

Third, EITR implicitly entails the assumption of complete information, which is the assumption that every individual not only knows its own payoffs but also knows the payoffs of all the other individuals in the group (and each individual knows everyone knows this, etc., so that payoffs are common knowledge). This seems to be an especially strong assumption. For example, in the example of immunization, it is reasonable to think that individuals do not perfectly know each others' preferences over tradeoff between health and other goods. In the example of educational achievement, it is reasonable to think that individuals do not perfectly know the ability of others in the group. EITR rules out incomplete information basically for the same reason that it rules out mixed strategies: if there is incomplete information then it is possible even in a Bayesian Nash equilibrium in which each type plays a pure strategy for the condition that $y_{ig} = \upsilon_{ig}(\mathbf{y}_{-ig}, \mathbf{d}_g)$ to be false, since with incomplete information each individual is effectively playing with individuals using mixed strategies induced by the distribution over types. This is in contrast to individualistic treatment response models in which uncertainty about the payoffs at the time of treatment choice is allowed since the econometrician observes the response to the realized treatment, regardless of how that treatment is selected.

Fourth, EGTR implicitly entails the assumption of a unique equilibrium even if there is pure strategy Nash equilibrium play. Otherwise, there are multiple potential outcomes for the group for any vector of treatments $\mathbf{d}$. And, indeed, in many settings of social interaction there are multiple equilibria according to reasonable solution concepts, like Nash equilibrium.

Consequently, in models with social interaction EITR (and/or EGTR) can imply significant assumptions on behavior that are not necessarily implied by economic theory. This claim is justified because EITR (and/or EGTR) can be false even if the economic model that generates the data implies the existence of best responses

[5]Mixed strategy equilibria do not necessarily mean that players need to randomize according to some probability. Rather, mixed strategies reflect a genuine indifference that players have about playing any outcome on their support: any of these actions yields the same payoff.

$\upsilon_{ig}(\cdot)$ (and/or $\upsilon_g(\cdot)$). In other words, the condition that individuals have best response functions $\upsilon_{ig}(\cdot)$ (and/or the group has a response correspondence $\upsilon_g(\cdot)$) by itself seems a very weak assumption, and may be justified by considering any of a variety of economic models for the social interaction under consideration. But, once the assumption of EITR (and/or EGTR) is added, implicitly additional strong assumptions are made on the underlying economic model that generates the data. In some settings EITR (and/or EGTR) is actually inconsistent with plausible economic models for the social interaction under consideration, as we will discuss.

However, there are a few settings beyond a unique pure strategy Nash equilibrium in a game with complete information in which EITR (and/or EGTR) may be satisfied. First, if there is perfect information, so that some individuals observe the decisions of the other individuals before making their own decisions, then it may be reasonable to assume EITR for some individuals. In individualistic treatment response models (i.e., models without social interaction) we can think of a game with perfect information in which "nature" selects the treatments, and then the individuals "respond" to that treatment. This justifies the implicit assumption of EITR in models without social interaction. Second, if it is known that there is a correlated equilibrium in which there is a publicly observed correlating device, and in which each individual plays a pure strategy as a function of the signal, then it may be reasonable to assume EITR.

Finally, note that neither of EITR and EGTR necessarily implies the other. EITR can hold while EGTR fails if there is an interaction with pure strategy Nash equilibrium play with complete information and multiple equilibria. EGTR can hold while EITR fails if there is a unique potential outcome according to some solution concept that describes the behavior of the group, but the outcome is not comprised of mutual best responses considered as pure strategies. For example, if there is incomplete information and if a given group has a fixed realization from the distribution of types, and behavior is described by a unique Bayesian Nash equilibrium with pure strategies for each type, then there is a unique potential outcome for the group so EGTR holds. But in general this does not comprise mutual best responses considered as pure strategies, so EITR fails. EITR and EGTR can both hold if there is a unique pure strategy Nash equilibrium with complete information. And both can fail if there are multiple potential outcomes that do not comprise mutual best responses considered as pure strategies, which may happen when the equilibrium is in mixed strategies with deliberate randomization.

In the rest of the paper we demonstrate this discussion by concrete examples of standard settings for models with social interaction. Section 3 studies an example of immunization and disease with social interaction. Section 4 studies the linear-in-means model in an educational achievement setting. In the conclusions we suggest two approaches in response to the failure of EITR and/or EGTR, and also discuss the wide applicability of these results to many sorts of data.

3. Immunization and disease with social interaction

Consider a model of immunization and disease in which individuals can make choices that affect their health. Such models are natural to consider in economics, but are unlike traditional epidemiology models. One such choice is the decision of whether to get immunized, and a related choice is the decision of whether, and how much, to interact with individuals who may have the disease under consideration.

We discuss EITR (and/or EIGR) in two related models of immunization and disease. The first is a model in which policymakers are interested in the *decision to get immunized*, whereby the object of interest is the best response function that gives the decision to immunize as a function of the immunization decisions by others in the group. The second model is a model of *health outcomes under some policy intervention*, whereby an individual's health outcome is a function of the vector of treatments of everyone in that individual's group. The object of interest in this second model is the health outcome production function for the group.

3.1. **Implications of EITR in models of the decision whether to get immunized.** Suppose we model the decision to get immunized for individual i in group g, y_{ig}, as being subject to a best response function $\upsilon_{ig}(\mathbf{y}_{-ig})$, and suppose we are trying to learn about functionals of $\upsilon_{ig}(.)$ using a data set $(\mathbf{y}_g : g = 1, \ldots, G)$ of group immunization decisions. We write the response function as only a function of the immunization decision of others, but this can be made implicitly to depend on exogenous observed and unobserved variables (i.e., $\upsilon_{ig}(\mathbf{y}_{-ig}) \equiv \upsilon(\mathbf{y}_{-ig}, X_{ig}, \epsilon_{ig})$). EITR assumes that for each group g, $y_{ig} = \upsilon_{ig}(\mathbf{y}_{-ig})$ for all individuals i.

To see why EITR rules out many plausible economic models for the decision whether to get immunized, consider for simplicity of exposition the immunization decisions of two people in close contact who take as given the immunization decisions of the rest of their community; the basic idea applies equally to larger groups, like immunization decisions of students in a classroom. Since the two people are in close contact it is very likely the immunization status of one individual affects the health outcomes of the other individual and thus that there is a social interaction in health outcomes and immunization status. There is a private cost to getting immunized (e.g., the monetary cost, the time cost, or the health cost of perceived negative effects of immunization), but a group benefit. Especially in some settings in developing countries, these costs can be relatively high. Agents may wish either to dis-coordinate or coordinate their immunization decisions, as described in the following two specifications.

Suppose that if at least one of the pair gets immunized they both "avoid" the illness and get β utils from health. However, there is a private cost of θ utils to getting the immunization. If neither gets immunized they both "get" the illness and get 0 utils from health. Assume that $0 < \theta < \beta$, and assume that these payoffs are common knowledge. This is described in game (a) in table 1, where action 0 is not get immunized and action 1 is get immunized. The set of Nash equilibria of this game are: two pure strategy equilibria in which one person gets immunized and the other doesn't, and a mixed strategy equilibrium in which both both people play a strategy to get randomly immunized with probability $\frac{\theta}{\beta}$. With these payoffs, there is a preference for un-coordination.[6]

Now suppose instead that both need to get immunized to avoid the illness. The payoffs are like in the case above, but modified to account for the fact that now both individuals need to get immunized for there to be a health benefit. This is described in game (b) in table 1. This is basically a coordination game where both players get the same payoff to coordinating, and the payoff depends on what they coordinate on, after a payoff normalization. The set of Nash equilibria of this game are: two pure strategy equilibria, one in which both individuals get immunized and one in which neither individual gets immunized, and a mixed strategy equilibrium in which both people play a strategy to get randomly immunized with probability $\frac{\beta-\theta}{\beta}$.

Similar results that suggest that immunization decisions involve randomization because of strategic considerations include, for example, Bauch, Galvani, and Earn (2003), Bauch and Earn (2004), Bauch, Bhattacharyya, Ball, and Boni (2010), Galvani, Reluga, and Chapman (2007), and Vardavas, Breban, and Blower (2007). In the

[6] This basically results in the payoffs of the game of chicken, as described in Fudenberg and Tirole (1991, p. 18) among many places, where the "weak" action corresponds to getting immunized and the "tough" action corresponds to not getting immunized.

Table 1. Two games for immunization decisions: (a) preference for dis-coordination (b) preference for coordination

(a)

	$y_2 = 0$	$y_2 = 1$
$y_1 = 0$	$0, 0$	$\beta, \beta - \theta$
$y_1 = 1$	$\beta - \theta, \beta$	$\beta - \theta, \beta - \theta$

(b)

	$y_2 = 0$	$y_2 = 1$
$y_1 = 0$	$0, 0$	$0, -\theta$
$y_1 = 1$	$-\theta, 0$	$\beta - \theta, \beta - \theta$

games described above the randomization is due to the possible use of mixed strategies, but there can also be randomization that is due to incomplete information, for example when individuals do not know the others' preferences over health and other goods. In that case, there is effectively randomization, from the perspective of each individual, induced by the distribution over types.

In addition to randomization, it is also plausible in this sort of setting that there is not Nash equilibrium play. For example, in the case of preference for un-coordination it is plausible that the outcome is that neither individual gets immunized, even though this is not a Nash equilibrium. This is plausible because it could be that each individual believes the other individual will get immunized, which is a reasonable belief since getting immunized is the best response if the other individual does not get immunized, and so on. In other words, neither individual getting immunized is a rationalizable outcome.

Consider data that consists of the immunization decisions of married couples.[7] The assumption of EITR implies that, for example, in a married couple in which both are observed to get immunized in the data, the best response of each to the other getting immunized is to get immunized itself. This rules out the possibility that the data comes from the mixed strategy equilibrium, or a rationalizable equilibrium. In both the case of Nash equilibrium in mixed strategies and non-Nash equilibrium play, like rationalizability, it need not be the case that immunization decisions $\mathbf{y}$ in the data imply that $y_{ig} = v_{ig}(\mathbf{y}_{-i})$. Consequently, in those cases, EITR does not hold.

In those cases, assuming EITR can negatively impact the validity of the conclusions that are drawn from data. Consider for example a public health organization

[7]It may seem more reasonable to model the immunization decisions of married couples as some sort of cooperative decision, but the basic idea holds, with more complicated notation, also in settings like a classroom, where a non-cooperative interaction is the preferred model.

that observes that fraction p of husbands whose wives are immunized also gets immunized himself, for some $p \in (0, 1)$. (Or vice versa.) Note that this is consistent with the mixed strategy Nash equilibrium behavior both when there is preference for dis-coordination and preference for coordination. The health organization might conclude from this, based on an implicit or explicit assumption of EITR, that it is worthwhile to promote immunization among women, with the "understanding" from the data that the husbands would be reasonably likely to also get immunized if their wives get immunized (assuming that p is reasonably large). However, the validity of this conclusion depends on whether there is preference for dis-coordination or preference for coordination. In the first case this policy intervention would have the result of reducing the rate of immunization among men to 0 while in the second case it would have the result of increasing the rate of immunization among men to 1. Obviously, these two scenarios have very different implications for the public health organization, but this sort of possibility is ruled out by assuming EITR. The same reasoning holds for other policy interventions. For example, a public health organization might consider whether to promote vaccination among some subpopulation of a school (e.g., a particular grade level, or the teachers and staff), using the observed data to help predict the behavioral response of the rest of the school. The same considerations imply that assuming EITR can negatively impact the validity of the conclusions the health organization draws from the data about such a policy intervention. It could be, for example, that the data suggests that the result of this policy intervention is that, again, roughly fraction p of the rest of the school would get immunized, because that is observed in the data, but as before this analysis rules out the possibility of mixed strategy Nash equilibrium or non-Nash equilibrium.

3.2. **Implications of EGTR in models of health outcomes.** Now suppose that the object of interest is the "production function" of health outcomes as a function of the vector of treatments in a group. The treatments could be provision or subsidization of immunization, supply of bed nets, some sort of public health awareness campaign, or anything else that a social planner can manipulate that affects health outcomes. Let the health outcome of individual i in group g, H_{ig}, be given by $H_{ig} = \tilde{v}_{ig}(\mathbf{d}_g, \mathbf{A}_g)$ where $\tilde{v}_{ig}(\cdot)$ is a function, and $\mathbf{A}_g$ is an unobservable that represents actions that individuals take, possibly in response to $\mathbf{d}_g$. For example, given the treatments in $\mathbf{d}_g$, $\mathbf{A}_g$ can capture the decision of individuals in the group of whether (and how much) to interact with other individuals who may have the disease. If

$\mathbf{A}_g$ is a *deterministic function* of $\mathbf{d}_g$ then it is without loss of generality to write that $H_{ig} = \upsilon_{ig}(\mathbf{d}_g) = \tilde{\upsilon}_{ig}(\mathbf{d}_g, \mathbf{A}_g(\mathbf{d}_g))$. Consequently EGTR is plausible under this condition, because then the group health outcome is a unique function of the treatment, even though there is an intermediate behavior stage that also affects health outcomes. For recent work on identification in these settings of treatment response where an outcome is a function of the treatments of others, see Lazzati (2010) and Manski (2010).

On the other hand, there are two important cases where $\mathbf{A}_g$ is not a unique function of $\mathbf{d}_g$. The first case is when the behavioral stage of the model, considered as conditional on the "parameters" $\mathbf{d}_g$, can have multiple equilibria. In this case, $\mathbf{A}_g$ can take many different values for some $\mathbf{d}_g$. As a consequence, it is not credible to assume EGTR, because for a fixed vector of treatments, the group can have multiple different potential outcomes. The second case is when the behavioral stage of the model involves mixed strategies with deliberate randomization. Then again, $\mathbf{A}_g$ can take many different values for some $\mathbf{d}_g$.

The possibility of models of behavior with mixed strategies has already been discussed above. For example, suppose that $\mathbf{d}_g$ amounts to some sort of intervention that affects θ, the private util cost of immunization, in the model of section 3.1. Then $\mathbf{A}_g$ is not a function. For example, one can think of $\mathbf{d}_g$ as a subsidy to get vaccinated (or buy a bed net in a malaria experiment, etc.), and $\mathbf{A}_g$ is some action that depends on $\mathbf{d}_g$ that affects payoffs, like whether to actually get immunized. In general, the health outcome differs depending on which actions are actually realized from the mixed strategies. For example, in both games, if it happens that both individuals get immunized, then neither individual gets the disease; but, if it happens that neither individual gets immunized, then both individuals get the disease. Consequently, under this model, there are multiple potential outcomes for the group given a fixed treatment $\mathbf{d}_g$. There are also multiple potential outcomes because of the existence of multiple equilibria in those games. For example, in the game with a preference for coordination, if it happens that the equilibrium in which both individuals get immunized is selected, then neither individual gets the disease; but, if it happens that the equilibrium in which neither individual gets immunized is selected, then both individuals get the disease.

Further, for another social interaction with multiple equilibria, consider models of disease in which individuals can modify their behavior (denoted above as $\mathbf{A}_g$) as a function of the prevalence of the disease. For example, in the case of a sexually transmitted infection (i.e., Kremer (1996)) this can be the number of sexual partners per period. More generally, the behavior can be the decisions relating to interaction with other individuals who may have the disease. As before, $\mathbf{d}_g$ is a vector of treatments like those discussed above, like subsidization of some sort of medical care. The actions $\mathbf{A}_g$ are the decisions of individuals about interacting with others. Kremer (1996) shows that these models can have multiple equilibria that have different prevalences of the disease. Consequently, the assumption of EGTR rules out this class of model of disease and behavior. The intuition for this result is described in detail in Kremer (1996).[8]

Even though the vector of treatments is exogenous to the model of health outcomes, the assumption of EGTR rules out models which have the potential for multiple equilibria. The assumption of EGTR also rules out mixed strategies with deliberate randomization.

4. Educational achievement with peer effects

In this section we focus on empirical models of classroom production, where the outcome is educational achievement (measured, for example, by test scores) and the social interaction is the possibility that the achievement of a student i depends endogenously on the achievement of the other students.

A typical econometric model in this literature (e.g., the linear-in-means model) simply assumes a reduced form in which the achievement of student i is a function of the achievement of the other students. These sorts of models do not make explicit the mechanism of the social interaction, so need not necessarily be logically consistent with any particular economic model of the interaction. Consequently, in order to determine whether the econometric model seems plausible it is useful to think about

[8]Roughly, the intuition is as follows. It can be an equilibrium for the prevalence to be relatively low, when people interact with relatively many other people. This is an equilibrium since the probability of interacting with someone who has the disease is relatively low in this case, so that optimally people interact with many people. The prevalence of the disease can remain low despite high levels of interaction when there is heterogeneity of individuals, so that enough people interact with relatively few people to still keep the prevalence low. Alternatively, it can be an equilibrium for the prevalence of the disease to be relatively high, when people interact with relatively few people. This is an equilibrium since the probability of interacting with someone who has the disease is relatively high in this case, so that optimally people interact with few people. The prevalence of the disease can remain relatively high despite low levels of interaction when there is heterogeneity of individuals, so that enough people interact with relatively many individuals to still keep the prevalence high. This intuition seems to apply to many sorts of models of health outcomes where there is a disease that can be transmitted between persons.

how the social interaction might work, using economic theory, and then think about how that relates to the econometric model.

One possible mechanism for the social interaction in the case of educational achievement is competition between the students. Students might be competing for grades assigned based on relative performance (e.g., "grading on a curve"), for permission to take subsequent classes that take only the highest achieving students (e.g., ability tracking), for interviews or jobs that depend on class rank (e.g., the labor market for business, law, medical, and perhaps other students in the United States and similar countries, or the labor market for government jobs in some countries), for letters of recommendations from the instructor, or simply for the psychic benefit of doing better than peers. This is related to the idea that education acts as a "filter" or "screen" for ability (i.e., Arrow (1973)).

This suggests that a plausible economic model for educational achievement in a classroom is that if student i has the k-th highest achievement, perhaps as measured by test score, it gets a reward of V_{ik} where $V_{i1} \geq V_{i2} \geq \cdots \geq 0$. (We can allow the valuations of the rewards to be symmetric or asymmetric across students. The valuations are symmetric if $V_{ik} = V_{jk}$ for all $i \neq j$ and all k, and are asymmetric otherwise. If the valuations are symmetric, the indexing by i is irrelevant. Also, when the valuations are asymmetric, the valuations can either be commonly known or private information.) The students must pay a cost of effort to produce educational achievement: the cost of educational achievement b_i for student i is $c_i(b_i)$ where $c_i(\cdot)$ is a cost function. This cost might reflect the psychic cost of learning, the opportunity cost of time in general, or, perhaps especially in developing countries, the opportunity cost of time as it relates to foregone wages or contribution to the household. Let $V_i(b_i, b_{-i})$ be the reward V_{ik} when b_i is the k-th highest achievement. Therefore the payoff to student i from educational achievement b_i when the other students have educational achievements b_{-i} is $V_i(b_i, b_{-i}) - c_i(b_i)$. Economic models like this are known variously as *all-pay auctions* or *contests*.

In general any Nash equilibrium of this interaction must involve randomization, either because there is complete information and students use mixed strategies or because there is incomplete information. The intuition for this result is evident in the case of two students and one reward with asymmetric valuation, with complete information and a shared cost function $c_i(b) = b$. Suppose that student 1 chooses educational achievement $b_1 > 0$ as a pure strategy. Student 2 has two possible "best responses:" either it selects an achievement of 0 and gets payoff 0 or it selects an achievement of $b_1 + \epsilon$ for $\epsilon > 0$ very small and gets a payoff of $V_2 - b_1 - \epsilon$.[9] In the former case student 1 has exerted too much effort and would prefer to deviate to an achievement of $0 + \epsilon$, so that it still wins V_1, but at a lower cost. In the latter case student 1 would prefer to deviate either to an achievement of 0, so that it still loses but exerts no effort, or to just out-achieve student 2's achievement of $b_1 + \epsilon$. This heuristic suggests that in general that there can be no Nash equilibrium in pure strategies in this sort of model.

Results like this have been proved formally under a variety of specifications: complete information with asymmetric valuations for one reward (i.e., Hillman and Riley (1989), Baye, Kovenock, and De Vries (1993), and Baye, Kovenock, and De Vries (1996)), complete information with asymmetric valuations for one reward under various types of upper bounds on the allowed achievement[10] (i.e., Che and Gale (1998), Kaplan and Wettstein (2006), and Kline (2009)), the equilibrium with complete information with symmetric valuations for many rewards (i.e., Barut and Kovenock (1998)), and the symmetric equilibrium with incomplete information about cost functions with symmetric valuations for many rewards (i.e., Moldovanu and Sela (2001)). A more general study of these types of models is Siegel (2009). We do not claim that any of these models are necessarily completely satisfactory as a model of competition in educational achievement. Rather, we simply make the case that it is a reasonably robust result that randomization is necessary in equilibrium in this type of model.

The use of randomization in equilibrium implies that the assumption of EITR is logically inconsistent with this class of economic model. The same reasoning implies that EGTR is logically inconsistent with this class of economic model, because mixed strategies with deliberate randomization implies that a given classroom can have many different potential outcomes.

4.1. **The linear-in-means reduced form model.** Consider the linear-in-means reduced form model for a classroom with $N + 1$ students, with $N \geq 1$, so that $y_i = \beta x_i + \frac{\alpha}{N} \sum_{j \neq i} y_j + \epsilon_i$. The parameters of this model are estimated from the observed data in many classrooms, where in each classroom the data is observations

[9] Note that student 2 does not technically have a best response to a pure strategy when $b_1 < V_1$. This is not a problem for us, exactly because we are showing that we should expect mixed strategies in equilibrium.

[10] This has been previously studied in the context of political lobbying, where the upper bound is understood to be a law that limits lobbying. In the context of educational achievement this upper bound might reflect the fact that there is an upper bound on "humanly possible" educational achievement, or a maximum possible test score.

of achievement, $\mathbf{y}$, and the exogenous observables, $\mathbf{x}$. Note that this model makes the assumption that in the data, the outcome of individual i is a function of the actual realized outcomes of $-i$ and that this relationship holds in the data.

One counterfactual policy experiment that motivates such a model is to determine what happens when some students are "assigned" to have particular educational achievements, in order to justify statements like that high achieving peers "cause" higher educational achievement. Of course, since the linear-in-means model involves the simultaneous determination of $\mathbf{y}$, the results of this counterfactual exercise cannot be determined directly based on the estimated parameters. Rather, such a counterfactual policy experiment is related to the best response function. The linear-in-means model cannot be the best response if the underlying social interaction follows the economic model considered above based on competition. This is because, if the linear-in-means model were the best response, and the model were estimated based on the observed data, then EITR would be implicitly assumed in order to relate the observed data (i.e., $\mathbf{y}$ and $\mathbf{x}$ in each classroom) to the object of interest (i.e., the best response function). And, as has been discussed, EITR would then rule out the economic model considered above.

A related approach would be to consider the effect of exogenously determined measures of ability, perhaps prior test scores, on educational achievement. According to this approach the object of interest is the effect of the exogenous ability of students other than i on the achievement of student i, which depends both on β and α. This implies that the object of interest is the group response function that is implied by the linear-in-means model that "solves out" the simultaneous determination of educational achievement, and so relates educational achievement to only the exogenous variables. However, this econometric model generically implies a unique outcome for the classroom,[11] as in the assumption of EGTR, which is also inconsistent with the

[11] This claim is established as follows. Let Υ be the $N+1 \times N+1$ matrix with 1s on the diagonal and $\frac{-\alpha}{N}$ in all off-diagonal elements. Let $\mathbf{d}$ be the $N+1 \times 1$ vector with ith row given by $\beta x_i + \epsilon_i$. Then stacked across students the linear-in-means model is equivalent to $\Upsilon \mathbf{y} = \mathbf{d}$. The determinant of Υ is $(1-\alpha)(1+\frac{\alpha}{N})^N$, so Υ is invertible as long as $\alpha \neq 1$ and $\alpha \neq -N$. So, except in those cases, $\mathbf{y} = \Upsilon^{-1}\mathbf{d}$, so that for any "treatment" of xs that enter $\mathbf{d}$, the model predicts a unique outcome $\mathbf{y}$ for the classroom. To put this closer to the notation used in the rest of this paper write $\mathbf{y} = \Upsilon^{-1}(\beta\mathbf{x} + \epsilon) \equiv \upsilon_g(\mathbf{d})$ where now $\mathbf{d} = \mathbf{x}$ and the ϵ is captured by indexing the group response correspondence by g. Under the assumption of sufficient variation in $\mathbf{x}$ (i.e., so that derivatives of the conditional expectation of $y_i|\mathbf{x}$ are observed) and the assumption that $E(\epsilon|\mathbf{x}) = 0$, the parameters $\Upsilon^{-1}\beta$ are point identified. The expectations (and sampling process) are with respect to classrooms. And, these parameters give the average marginal effect of exogenous observables on educational outcomes, according to the group response function.

economic models above involving mixed strategies with deliberate randomization, or other economic models in which there are multiple equilibria.

4.2. **A Monte Carlo experiment.** Consider a game of incomplete information in which the utility of player i of type θ_i is $u_i(a_i, a_{-i}; \theta_i) = \theta_i a_i - \frac{1+\Phi}{2} a_i^2 + \Phi \frac{a_i}{N} \sum_{j \neq i} a_j$. The type parameters θ_i are private information, and are iid within a group. Then, integrating out player i's uncertainty about a_j, the utility of player i is $\theta_i a_i - \frac{1+\Phi}{2} a_i^2 + \Phi \frac{a_i}{N} \sum_{j \neq i} E(a_j)$. There is a Bayesian Nash equilibrium in which $E(a) = E(\theta)$, and $a_i(\theta_i) = \frac{\theta_i}{1+\Phi} + \frac{\Phi}{1+\Phi} E(a)$. This can motivate the linear-in-means reduced form as the strategy function of the players, as a function of their type. (But not that in general this is different than the interpretation as the best response function, because here the action is a function of beliefs about actions, whereas with the best response it is a function of actual actions. However, given the linear form of this utility function, it turns out that in this special case, it is trivial to translate from one interpretation to the other, so that Φ also characterizes the best response function.) In particular, suppose that $\theta_i = X_i \beta + \alpha Z_g + \epsilon_i$ where X_i is observed by the econometrician while ϵ_i and Z_g is unobserved. Then this economic model says that the observed action of player i in the data is $\frac{X_i \beta + \alpha Z_g + \epsilon_i}{1+\Phi} + \frac{\Phi}{1+\Phi} E(a)$. The objects of interest are the unknown parameters β and Φ. Note that the beliefs $E(a)$ are not known by the econometrician, so estimating this model is not straightforward. One possibility is to use "standard" techniques used to estimate the linear-in-means model of $y_i = \frac{X_i \beta + \alpha Z_g + \epsilon_i}{1+\Phi} + \frac{\Phi}{1+\Phi} \frac{1}{N} \sum_{j \neq i} y_j$. This appears to be a closely related model, where the only difference is replacing the beliefs $E(a)$ with the sample average $\frac{1}{N} \sum_{j \neq i} a_j$. This might be "justified" on the basis that $\frac{1}{N} \sum_{j \neq i} a_j$ is the "sample analog" of $E(a)$. This Monte Carlo experiment demonstrates that if these "standard" techniques are used to estimate this model, the estimates do not recover the true parameters. On the other hand, if the data is actually generated by the linear-in-means model, as expected the standard techniques do recover the true parameters. This shows that the linear-in-means model (at least as estimated by standard techniques) is not a good approximation when the data is actually generated by a game.

Details of the two DGPs:

The first steps of the two DGPs are the same. For each group, a random variable Z_g is drawn. Also, for each individual in each group, random variables X_{1ig}, X_{2ig}, and ϵ_{ig} are drawn. These random variables are all distributed $N(0, 2)$ and are all independent (across groups, across individuals, etc.). Then the type of each player is

	Size of groups		
Number of groups	10	25	100
10	0.202	0.200	0.190
25	0.197	0.201	0.189
100	0.199	0.198	0.195
500	0.200	0.199	0.199

TABLE 2. Data is generated from the linear in means model

computed as $\theta_{ig} = \beta X_{ig} + \alpha Z_g + \epsilon_{ig}$, where β and α are specified. In particular, the β parameters are 1 and α is 0.05. And, $\Phi = 0.25$.

Then, for the game specification, the equilibrium beliefs of each player in group g are that $E(a) = E(\theta) = \alpha Z_g$. Then the actions are specified to be according to the strategy function derived above, so that $a_{ig} = \frac{X_{ig}\beta + \alpha Z_g + \epsilon_{ig}}{1+\Phi} + \frac{\Phi}{1+\Phi}\alpha Z_g$.

Alternatively, for the linear in means specification, the data is generated by stacking $X_{ig}\beta + \alpha Z_g + \epsilon_{ig}$ over i, to construct a vector θ_g, and then specifying that $a_g = \Upsilon^{-1}\frac{\theta_g}{1+\Phi}$, where Υ is appropriately specified so that the data comes directly from the system of simultaneous equations. (This is just algebra from the equations that $y_i = \frac{\theta_i}{1+\Phi} + \frac{\Phi}{1+\Phi}\frac{1}{N}\sum_{j\neq i} y_j$.)

The estimator for both DGPs is the same, and is the systems 2SLS estimator that is the "obvious" estimator based on thinking of the linear in means model as a system of simultaneous equations for each group. The x's serve as instruments for the endogenous rhs variables in this estimator.

The results are summarized in the tables. The estimates in these tables are of the "parameter" $\frac{\Phi}{1+\Phi}$, because that is how Φ shows up in the model, so that since $\Phi = 0.25$, the "truth" in the following table is 0.20. In the first table, the data generating process is the linear in means model, and the estimates are basically correct. In the second table, the data generating process is the game, and the results seem to be basically 0, and thus incorrect.

5. Conclusion and discussion

We have studied two common but previously unaddressed assumptions that we call empirical individual treatment response and empirical group treatment response. We show that these extensions of standard assumptions in models of individualistic

	Size of groups		
Number of groups	10	25	100
10	0.004	-0.001	-0.008
25	-0.004	0.001	-0.014
100	-0.001	-0.003	-0.006
500	-0.000	-0.001	-0.001

TABLE 3. Data is generated from the game

treatment response entail hidden behavioral assumptions when applied to settings with social interaction. We conclude with two remarks. The first is about how possibly to proceed in response to the conclusions of this paper, and the second is about the applicability of the conclusions to experimental data.

5.1. **Alternative models of interaction.** There are at least two possible responses to the fact that EITR and EGTR entail hidden behavioral assumptions when applied to settings with social interaction.

First, it is possible to conclude that the behavioral assumptions that are implied by EITR and/or EGTR are credible, and if so to maintain EITR and/or EGTR and proceed as usual. In some cases this means ruling out a class of plausible economic model, especially when there is incomplete information or the use of mixed strategies, as in the example of competition in educational achievement.

A related approach when the object of interest is some functional of the group response $\upsilon_g(\mathbf{d}_g)$ is to observe that even when EGTR fails it may still be the case that the *distribution* of the group responses are observed. For example, suppose that the underlying game involves mixed strategies with deliberate randomization. As we have shown, this implies that $\upsilon_g(\mathbf{d}_g)$ can be a correspondence rather than a function, in which case EGTR fails. However, it can be reasonable to assume that by observing a population of groups it still is true that the *distribution* of outcomes induced by the mixed strategy is observed. However, this approach still requires abandoning the assumption of EGTR, so that not all individualistic treatment response models translate to models with social interaction.

Similarly, it can be reasonable to assume that when there are multiple equilibria it still is true that the *distribution* of outcomes induced by the mechanism that selects the equilibrium is observed. However, this assumption may be slightly less credible, as economic theory typically does not pin down the selection mechanism. If it is the case that the selection mechanism accounts for factors that affect the outcome of the interaction that are not explicitly modeled and are unlikely to be stable features of the social interaction, as in the sunspot interpretation of a correlated equilibrium, then it may be appropriate to be reluctant to assume that the selection mechanism observed in the data is necessarily also the selection mechanism that would arise during whatever counterfactual policy experiment is conducted. There is relatively less reason to assume that the data provides useful information about selection, compared to information about mixed strategies, since any time the interaction "happens" when examining some counterfactual policy effect, the selection mechanism need not necessarily be the previously "estimated" selection mechanism, under the "sunspot" interpretation. In contrast, the mixed strategy is a stable feature of the interaction. This is somewhat similar to the Lucas (1976) critique of macroeconometric modeling: since the selection model is not pinned down by economic theory, there is reason to be skeptical it is a stable feature of the social interaction that would not change in response to a policy intervention, whereas the set of equilibria (and best responses, etc.) are pinned down by economic theory, providing a basis for them to be a stable feature of the social interaction. See for example Kline and Tamer (2010) on the identification of best responses in this sort of setting.

Another related approach is to maintain that even when EITR and/or EGTR fails, that they hold approximately. In this way, for example, it might be assumed even though a group has multiple potential outcomes because of the use of mixed strategies with deliberate randomization, or the existence of multiple equilibria, that all of these equilibria have basically the same outcome, so that EGTR holds approximately. If the differences in outcomes are sufficiently small, then it may be reasonable to still assume EGTR in the analysis. But, as with all of the results in this paper, establishing the credibility of this approach requires considering the economic model that generates the data from the underlying social interaction.

Second, it is possible to use a model without making these assumptions. A model that relaxes EITR and/or EGTR need not necessarily involve relaxing all of the behavioral assumptions we have discussed here. For example, it is possible to consider a model that rules out incomplete information, but allows mixed strategies. The most closely related paper in this literature is Kline and Tamer (2010), which studies the partial identification of best responses in binary games, under a variety of behavioral assumptions. Other papers like Bresnahan and Reiss (1991), Tamer (2003), Ciliberto and Tamer (2009), Bajari, Hong, and Ryan (2010), and Bajari, Hong, Krainer, and Nekipelov (2010) study identification of the payoffs of a game, which is stronger than identification of best responses.

5.2. **Applicability of these conclusions to experimental data.** These conclusions apply equally to observational data and experimental data. There are many advantages of experiments in areas like development and labor economics, where, these experiments might imply that it is not necessary to use sophisticated econometric modeling, for example because experiments can solve endogeneity problems. Nevertheless, the conclusions of this paper suggest that, despite those advantages, there is still significant scope for econometric modeling of the data from experiments when there is social interaction. Experimental data and econometric modeling of social interaction based on economic theory provide complementary features to an analysis: the experiment helps to solve issues with endogeneity with respect to the relation between observable and unobservable characteristics of the individuals, while the econometric modeling helps resolve the issues discussed in this paper, that are unrelated to the endogeneity problem, but are related to how to link data and models in settings where social interactions are important.

References

Arrow, K. (1973): "Higher education as a filter," *Journal of Public Economics*, 2(3), 193–216.

Bajari, P., H. Hong, J. Krainer, and D. Nekipelov (2010): "Estimating static models of strategic interactions," *Journal of Business and Economic Statistics*, 28(4), 469–482.

Bajari, P., H. Hong, and S. P. Ryan (2010): "Identification and estimation of a discrete game of complete information," *Econometrica*, 78(5), 1529–1568.

Barut, Y., and D. Kovenock (1998): "The symmetric multiple prize all-pay auction with complete information," *European Journal of Political Economy*, 14(4), 627–644.

Bauch, C., S. Bhattacharyya, R. Ball, and M. Boni (2010): "Rapid emergence of free-riding behavior in new pediatric immunization programs," *PLoS ONE*, 5(9), 505–514.

Bauch, C., and D. Earn (2004): "Vaccination and the theory of games," *Proceedings of the National Academy of Sciences of the United States of America*, 101(36), 13391.

Bauch, C., A. Galvani, and D. Earn (2003): "Group interest versus self-interest in smallpox vaccination policy," *Proceedings of the National Academy of Sciences of the United States of America*, 100(18), 10564.

Baye, M., D. Kovenock, and C. De Vries (1993): "Rigging the lobbying process: an application of the all-pay auction," *American Economic Review*, 83(1), 289–294.

——— (1996): "The all-pay auction with complete information," *Economic Theory*, 8(2), 291–305.

Bernheim, B. (1984): "Rationalizable strategic behavior," *Econometrica*, 52(4), 1007–1028.

Blume, L., W. A. Brock, S. N. Durlauf, and Y. Ioannides (2010): "Identification of Social Interactions," in *Handbook of Social Economics, forthcoming.*

Bramoulle, Y., H. Djebbari, and B. Fortin (2009): "Identification of peer effects through social networks," *Journal of Econometrics*, 150(1), 41 – 55.

Bresnahan, T., and P. Reiss (1991): "Empirical models of discrete games," *Journal of Econometrics*, 48(1-2), 57–81.

Brock, W. A., and S. N. Durlauf (2001): "Discrete choice with social interactions," *The Review of Economic Studies*, 68(2), 235–260.

Camerer, C., T. Ho, and J. Chong (2004): "A cognitive hierarchy model of games," *Quarterly Journal of Economics*, 119(3), 861–898.

Case, A. C. (1991): "Spatial patterns in household demand," *Econometrica*, 59(4), 953–965.

Che, Y., and I. Gale (1998): "Caps on political lobbying," *American Economic Review*, 88(3), 643–651.

Ciliberto, F., and E. Tamer (2009): “Market structure and multiple equilibria in airline markets,” *Econometrica*, 77, 1791–1828.

Cliff, A. D., and J. K. Ord (1975): “Space-time modelling with an application to regional forecasting,” *Transactions of the Institute of British Geographers*, (64), 119–128.

Costa-Gomes, M., and V. Crawford (2006): “Cognition and behavior in two-person guessing games: an experimental study,” *American Economic Review*, 96(5), 1737–1768.

Fudenberg, D., and J. Tirole (1991): *Game Theory*. MIT Press.

Galvani, A., T. Reluga, and G. Chapman (2007): “Long-standing influenza vaccination policy is in accord with individual self-interest but not with the utilitarian optimum,” *Proceedings of the National Academy of Sciences of the United States of America*, 104(13), 5692.

Graham, B. S. (2008): “Identifying social interactions through conditional variance restrictions,” *Econometrica*, 76(3), 643–660.

Hillman, A., and J. Riley (1989): “Politically contestable rents and transfers,” *Economics & Politics*, 1(1), 17–39.

Jackson, M. (2010): *Social and Economic Networks*. Princeton University Press.

Kaplan, T., and D. Wettstein (2006): “Caps on political lobbying: Comment,” *American Economic Review*, pp. 1351–1354.

Kline, B. (2009): “A restriction on lobbyist donations,” *Economics Letters*, 104(3), 129–132.

Kline, B., and E. Tamer (2010): “Bounds for best response functions in binary games,” forthcoming in *Journal of Econometrics*.

Krauth, B. V. (2006): “Simulation-based estimation of peer effects,” *Journal of Econometrics*, 133(1), 243 – 271.

Kremer, M. (1996): “Integrating behavioral choice into epidemiological models of AIDS,” *The Quarterly Journal of Economics*, 111, 549–573.

Lazzati, N. (2010): “Treatment response with social interactions: partial identification via monotone comparative statics,” Working Paper.

Lee, L.-F. (2004): “Asymptotic distributions of quasi-maximum likelihood estimators for spatial autoregressive models,” *Econometrica*, 72(6), 1899–1925.

Lee, L.-f., X. Liu, and X. Lin (2010): “Specification and estimation of social interaction models with network structures,” *Econometrics Journal*, 13(2), 145–176.

Lucas, R. (1976): “Econometric policy evaluation: a critique,” *Carnegie-Rochester Conference Series on Public Policy*, 1(1), 19–46.

MANSKI, C. (1993): "Identification of endogenous social effects: the reflection problem," *The Review of Economic Studies*, 60(3), 531–542.

MANSKI, C. (2010): "Identification of treatment response with social interactions," Northwestern University Working Paper.

MCKELVEY, R., AND T. PALFREY (1995): "Quantal response equilibria for normal form games," *Games and Economic Behavior*, 10(1), 6–38.

MOLDOVANU, B., AND A. SELA (2001): "The optimal allocation of prizes in contests," *American Economic Review*, 91(3), 542–558.

PEARCE, D. (1984): "Rationalizable strategic behavior and the problem of perfection," *Econometrica*, 52(4), 1029–1050.

SACERDOTE, B. (2001): "Peer effects with random assignment: results for Dartmouth roommates," *The Quarterly Journal of Economics*, 116(2), 681–704.

SIEGEL, R. (2009): "All-pay contests," *Econometrica*, 77(1), 71–92.

TAMER, E. (2003): "Incomplete simultaneous discrete response model with multiple equilibria," *Review of Economic Studies*, 70(1), 147–165.

VARDAVAS, R., R. BREBAN, AND S. BLOWER (2007): "Can influenza epidemics be prevented by voluntary vaccination," *PLoS Computational Biology*, 3(5), e85.