Corrigendum on

Incomplete Bivariate Discrete Response Model with Multiple Equilibria

Elie Tamer

Princeton University

January 19, 2004

Lemma 6 on page 163 of Tamer (2003) (TA) contains an error[1] that resulted in a a wrong formula for the asymptotic variance Ω_{SML} of the semiparametric maximum likelihood estimator (SML) provided on page 158. I provide a correct formula for this variance. Needless to say, this note is not self contained. The notation used here is the same as the one in TA so this note should be read together with Tamer (2003).

Starting from page 162, we have (setting $1_{(y_{i1},y_{i2})}(1,0) = 1_{\mathbf{y_i}}(1,0)$)

$$\begin{aligned}
\Gamma(\hat{H}_n) =& \frac{1}{N}\sum_{i=1}^{N}\left(\frac{(\partial_\beta P_{1i} + \partial_\beta P_{2i})}{(1 - P_{1i} - P_{2i} - \hat{H}_{ni})} - \frac{(\partial_\beta P_{1i} + \partial_\beta P_{2i})}{(1 - P_{1i} - P_{2i} - H_i)}\right) 1_{\mathbf{y_i}}(1,0) \\
=& \frac{1}{N}\sum_{i=1}^{N}\left(\frac{(\partial_\beta P_{1i} + \partial_\beta P_{2i})}{(1 - P_{1i} - P_{2i} - H_i)} - \frac{(\partial_\beta P_{1i} + \partial_\beta P_{2i})\,(1 - P_{1i} - P_{2i})}{(1 - P_{1i} - P_{2i} - H_i)^2}\right) \frac{(\hat{f}_i - f_i)}{f_i} 1_{\mathbf{y_i}}(1,0) \\
&+ \frac{1}{N}\sum_{i=1}^{N}\left(\frac{(\partial_\beta P_{1i} + \partial_\beta P_{2i})}{(1 - P_{1i} - P_{2i} - H_i)^2}\right) \frac{(\hat{m}_i - m_i)}{f_i} 1_{\mathbf{y_i}}(1,0) + \text{rem} \\
=& \frac{1}{N}\sum_{i=1}^{N}\frac{(\partial_\beta P_{1i} + \partial_\beta P_{2i})}{(1 - P_{1i} - P_{2i} - H_i)^2}\left(\frac{(\hat{m}_i - m_i)}{f_i} - \frac{H_i(\hat{f}_i - f_i)}{f_i}\right) 1_{\mathbf{y_i}}(1,0) + \text{rem} \\
=& \frac{1}{N}\sum_{i=1}^{N} G_i(\beta)\left(\frac{\hat{m}_i - m_i}{f_i} - \frac{H_i(\hat{f}_i - f_i)}{f_i}\right) 1_{\mathbf{y_i}}(1,0) + \text{rem} \\
=& \frac{1}{N^2}\sum_{i,j}\frac{G_i(\beta)}{f_i}\left(\frac{1_{\mathbf{y_j}}(0,1)}{h^d}k(\frac{x_i - x_j}{h}) - m_i\right) 1_{\mathbf{y_i}}(1,0) - \frac{G_i(\beta)H_i}{f_i}\left(\frac{1}{h^d}k(\frac{x_i - x_j}{h}) - f_i\right) 1_{\mathbf{y_i}}(1,0) + \text{rem} \\
=& \frac{1}{N^2}\sum_{i,j} m_1(z_i, z_j) + \frac{1}{N^2}\sum_{i,j} m_2(z_i, z_j) + \text{rem}
\end{aligned}$$

For the main terms, using U-statistics calculations similar to the ones in Powell, Stock, and Stoker

[1] I would like to thank J. Hahn for pointing this out to me.

(1989) we get that first the bias is of order $o(\frac{1}{\sqrt{n}})$ by assumption 6 in TA since for example

$$Em_1(z_i,z_j) = \int \frac{G_i(\beta)}{f_i}\left(\int \frac{1}{h^d}1_{(y_{j1},y_{j2})}(0,1)k(\frac{x_i-x_j}{h})dF(z_j) - m_i\right)(1-P_{1i}-P_{2i}-H_i)dF(z_i)$$

Similar calculations show that $E(m_1(z_i,z_j)|z_i) = o(\frac{1}{\sqrt{n}})$ and $E(m_2(z_i,z_j)|z_i) = o(\frac{1}{\sqrt{n}})$. Also, lemma 3.11 of Powell, Stock, and Stoker (1989) can be easily verified by noting that $Em_1^2(z_i,z_j) = O(\frac{1}{h^d})$ and $Nh^d \to \infty$ by lemma 7 of TA. Now, applying the projection formula for U-statistics, we get that up to $o_p(1)$ terms $\sqrt{N}\,\Gamma(\hat{H}_n)$ is approximated by

$$\begin{aligned}\sqrt{N}\Gamma(\hat{H}_n) &= \frac{1}{\sqrt{N}}\sum_{i=1}^{N} -(\partial P_{1i}+\partial P_{2i})\frac{1_{(y_{i1},y_{i2})}(0,1)-H_i}{1-P_{1i}-P_{2i}-H_i} + \sqrt{N}\text{rem}\\ &= -\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(\partial P_{1i}+\partial P_{2i})\frac{1_{(y_{i1},y_{i2})}(0,1)-H_i}{1-P_{1i}-P_{2i}-H_i} + o_p(1)\end{aligned}$$

where the last equality follows from lemma 7 of TA. The above shows that the argument in the lemma 6 of TA is not correct and that $\sqrt{N}\Gamma(\hat{H}_n)$ is not or order $o_p(1)$ asymptotically. Hence, the "Normality Proof" in the middle of page 164 should read

$$\begin{aligned}\sqrt{n}(\hat{\beta}-\beta) = \left[-\partial^2 L(\beta;H)\right]^{-1}\frac{1}{\sqrt{n}}\sum_{i=1}^{N}\Bigg[&\frac{\partial P_{1i}}{P_{1i}}1_{(y_{i1},y_{i2})}(1,1) + \frac{\partial P_{2i}}{P_{2i}}1_{(y_{i1},y_{i2})}(0,0)\\ &- \frac{(\partial P_{1i}+\partial P_{2i})}{(1-P_{1i}-P_{2i}-\hat{H}_{ni})}1_{(y_{i1},y_{i2})}(1,0)\\ &- (\partial P_{1i}+\partial P_{2i})\frac{1_{(y_{i1},y_{i2})}(0,1)-H_i}{1-P_{1i}-P_{2i}-H_i}\Bigg] + o_p(1)\end{aligned}$$

The asymptotic distribution of the SML estimator is

$$\sqrt{N}(\hat{\beta}-\beta) \to \mathcal{N}(0, \Omega_{SML} = A^{-1}BA^{-1})$$

where

$$\begin{aligned}A &= E\left\{\left[\frac{\partial P_1\partial P_1'}{P_1} + \frac{\partial P_2\partial P_2'}{P_2} + \frac{(\partial P_1+\partial P_2)(\partial P_1+\partial P_2)'}{1-P_1-P_2-H}\right]\right\}\\ B &= E\left[\frac{\partial P_1\partial P_1'}{P_1} + \frac{\partial P_2\partial P_2'}{P_2} + \frac{(\partial P_1+\partial P_2)(\partial P_1+\partial P_2)'}{1-P_1-P_2-H} + \frac{(\partial P_1+\partial P_2)(\partial P_1+\partial P_2)'}{(1-P_1-P_2-H)^2}H(1-H)\right]\end{aligned}$$

The last term in B, $E\frac{(\partial P_1+\partial P_2)(\partial P_1+\partial P_2)'}{1-P_1-P_2-H}H(1-H)$ is not zero and the SML is not necessarily more efficient than the ML estimator defined on 157 of TA. This means that the conclusion of Theorem 3 is not in general correct. A minimum distance estimator is a theoretically more attractive estimator that can be used in models with inequality restrictions on regressions. Not only can this estimator be used to estimate β in (4) (page 153 in TA), but also β in (5) where the ML estimator in (7) cannot be used. The asymptotic properties of a similar minimum distance based estimator was studied in Hong and Tamer (2003).

References

Hong, H., and E. Tamer (2003): "Inference in Censored Models with Endogenous Regressors," *Econometrica*, 71(3), 905–932.

Powell, J., J. Stock, and T. Stoker (1989): "Semiparametric Estimation of Index Coefficients," *Econometrica*, pp. 1403–1430.

Tamer, E. T. (2003): "Incomplete Bivariate Discrete Response Model with Multiple Equilibria," *Review of Economic Studies*, 70, 147–167.